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|  |  | itbhopal.ac.in |  |
|  | TERM END EXAMINA | (TEE)-DEC-202 |  |
| Programme | B.Tech. (All Branches) | Semester | Fall 2021-22 |
| Course Name | Calculus and Laplace Transforms | Course Code | MAT1001 |
| Faculty Name | Dr. Manisha Jain | Slot / Class No | $\begin{aligned} & \text { A21+A22+A23 } \\ & \text { BL2021221000125 } \end{aligned}$ |
| Time | 1.5 Hrs. | Max. Marks | 50 |

## Answer ALL the Questions

| $\begin{gathered} \text { Q. } \\ \text { No. } \end{gathered}$ |  | Question Description | Marks |
| :---: | :---: | :---: | :---: |
| PART - A - ( $\mathbf{3 \times 1 0 = 3 0}$ Marks) |  |  |  |
| 1 | (a) | Find the Directional Derivative of scalar function $f(x, y, z)=\sqrt{x y z}$ at the point $\mathrm{A}(2,2,3)$ in the direction of the outward drawn normal of the surface of the sphere having radius 6 cm through the point $P$ | 10 |
|  | OR |  |  |
|  | (b) | Evaluate the Integral $\int_{0}^{1} \int_{y \sqrt{3}}^{\sqrt{4-y^{2}}} \sqrt{x^{2}+y^{2}} d x d y$ <br> (1) Draw the region <br> (2) High light all important points | 10 |
| 2 | (a) | Verify Gauss Divergent Theorem $\overline{A=} 4 x i-2 y^{2} j+z^{2} k$ taken over the region bounded by $x^{2}+y^{2}=4, z=0 \text { and } z=3$ | 10 |
|  | OR |  |  |
|  | (b) | If $\bar{A}=\left(2 x^{2}-3 z\right) i-2 x y j-4 x k$ and V is the closed region bounded by the planes $x=0, y=0$ and $2 x+2 y+z=4$ evaluate $\iiint(\Delta \times \bar{A}) d V$ | 10 |


| 3 | (a) | Solve the differential equation by using variation of parameters method (Write and highlights all important results) $\left(D^{2}+2 D+2\right) y=e^{-x} \sec ^{3} x$ | 10 |
| :---: | :---: | :---: | :---: |
|  | OR |  |  |
|  | (b) | Solve the following differential equation by using Laplace Transformation $\frac{d^{2} x}{d x^{2}}+5 \frac{d x}{d x}+6 x=5 e^{t} ; x(0)=2 ; x^{\prime}(0)=1$ | 10 |
|  | Part - B - $\mathbf{2} \times 10=20$ Marks $)$ |  |  |
| 4 |  | Calculate the integral $\int_{2}^{4} \int_{0}^{x+y} \int_{0}^{x+y} z d x d y d z$ <br> i. Describe the functions properly <br> ii. Draw the figure <br> iii. Correct the order of integration if required | 10 |
| 5 | (a) (b) | Solve the differential equation $\left[x \tan \left(\frac{y}{x}\right)-y \sec ^{2}\left(\frac{y}{x}\right)\right] d x+x \sec ^{2}\left(\frac{y}{x}\right) d y=0$ <br> By using Laplace Transform find show that $\begin{aligned} & \int_{0}^{\infty} e^{-s t} t^{3} \sin t d t=\frac{24 s\left(s^{2}-1\right)}{\left(s^{2}+1\right)^{4}} \\ & \text { Hence evaluate } \int_{0}^{\infty} e^{-t} t^{3} \sin t d t \end{aligned}$ | 10 |

