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TERM END EXAMINATIONS (TEE) – December 2021-January 2022

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|---------------------|--|------------------------|-------------------------|
| Programme | B.Tech | Semester | Fall 2021-2022 |
| Course Name | Calculus and Laplace transforms | Course Code | MAT1001 |
| Faculty Name | Dr. A.Manickam | Slot / Class No | A21+A22+A23/0164 |
| Time | 1½ hours | Max. Marks | 50 |

Answer ALL the Questions

| Q. No. | Question Description | Marks |
|---------------------------------------|---|-----------|
| PART - A – (3 x 10 = 30 Marks) | | |
| 1 | (a) Prove that the rectangular solid of maximum volume which is inscribed in a sphere is a cube. | 10 |
| | OR | |
| | (b) Evaluate $\iiint dzdydx$, where V is the finite region of space (tetrahedron) formed by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$ | 10 |
| 2 | (a) Verify the Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ over the cube bounded by $x = 0, x = a, y = 0, y = b, z = 0$ & $z = c$. | 10 |
| | OR | |
| | (b) Find $\int_C \vec{F} \cdot d\vec{r}$, if $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ moves a particle from (0,0,0) to the point (1,1,1) along $x = t, y = t^2, z = t^3$ | 10 |
| 3 | (a) Solve by using method of Cauchy's Legendre's linear differential Equations $(x + 2)^2 \frac{d^2y}{dx^2} - (x + 2) \frac{dy}{dx} + y = 3x + 4$ | 10 |
| | OR | |

| | | |
|-----|---|-----------|
| (b) | <p>(i). Find the Laplace Transform of the functions</p> $\frac{e^{-2t} - e^{3t}}{t}$ <p>(ii) Using Laplace transform to find the value of the definite integral for the following functions</p> $\int_0^{\infty} e^{-2t} t \cos t dt$ | 10 |
|-----|---|-----------|

Part - B – (2 x 10 = 20 Marks)

| | | |
|---|--|-----------|
| 4 | <p>(i). The temperature at a point (x, y, z) in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at $(4, 4, 2)$ desires to fly in such a direction that it gets cooled faster. Find the direction in which it should fly?</p> <p>(ii) In what direction from $(3, 1, -2)$ is the directional derivative of $\varphi = x^2 y^2 z^4$ maximum.</p> | 10 |
| 5 | Solve $y'' - 3y' + 2y = 4e^{-t}$, given that $y(0) = 2, y'(0) = 3$, by using Laplace transform Techniques. | 10 |

