Reg. No.:
Name :


TERM END EXAMINATIONS (TEE) - December 2021-January 2022

| Programme | B.Tech | Semester | Fall 2021-2022 |
| :--- | :--- | :--- | :--- |
| Course Name | Calculus and Laplace Transform | Course Code | MAT1001 |
| Faculty Name | Dr. Anant Kant Shukla | Slot / Class No | C11+C12+C13/BL2021 <br> $\mathbf{2 2 1 0 0 0 1 3 2}$ |
| Time | $11 / 2$ hours | Max. Marks | $\mathbf{5 0}$ |

## Answer ALL the Questions

Q. No.

## Question Description

Marks
PART - A-( $\mathbf{3} \times 10=30$ Marks $)$
1 (a) Find the directional derivative of the scalar function $f(x, y)=x^{2} y^{3}+x y-5$, in the direction of a unit vector which makes an angle of $30^{\circ}$ with the positive $x$-axis in the $X Y$-palne.
(b) Find the value of the double integral $\int_{0}^{1} \int_{x^{2}}^{2-x} x y d y d x$, by changing the order of integration.
(a) By using Gauss-Divergence theorem find the value of the surface integral $\iint_{S} \vec{F} \cdot \vec{n} d S$, where $\vec{F}=\left[8 x,-2 y^{2}, z^{2}\right]$ and $S$ is $x^{2}+y^{2}=8, z=0, z=3$.

OR
(b) Solve the differential equation $y^{\prime \prime}(t)+y^{\prime}(t)-2 y(t)=1-2 t, y(0)=0, y^{\prime}(0)=4$ by using the Laplace transform.
(a) Check whether the function $e^{3 x}$ is an integrating factor for the differential equation $\left(3 x^{2} y+2 x y+y^{3}\right) d x+\left(x^{2}+y^{2}\right) d y=0$. If yes, then find the respective exact differential equation and its solution when $y(0)=0$.
OR
(b) Let $f(t)=\left\{\begin{array}{l}1, \quad 0 \leq t<1 \\ 2-t, 1 \leq t \leq 2\end{array}\right.$ be a periodic function with period $T=2$. Find the Laplace transform of $f(t)$.

$$
\text { Part - B - }(2 \times 10=20 \text { Marks })
$$

Evaluate the line integral $\int_{C} \vec{F}$. $d \vec{r}$, where $\vec{F}=\left[x y, x^{2}+y^{2}\right]$ and $C$ is the $x$-axis from $x=2$ to $x=4$ and the line from $y=0$ to $y=12$.
5 Find the solution of the differential equation $y^{\prime \prime}(x)+8 y^{\prime}(x)+16 y(x)=5 e^{-4 x}$ by the method of undetermined coefficients.

