Reg. No.:	
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TERM END EXAMINATIONS (TEE) – December 2021- January 2022

Programme	B.Tech	Semester	Fall 2021-2022
Course Name	Calculus and Laplace Transform	Course Code	MAT1001
Faculty Name	Dr. Navneet Kumar Verma	Slot / Class No	(C11+C12+C13)/0131
Time	1½ hours	Max. Marks	50

Answer ALL the Questions

Q. No.	Question Description	Marks
	PART - A (30 Marks)	
1 **(a)**	A tree trunk of length l metres has the shape of a frustum of a circular cone with radii of its ends $a\ and\ b$ metres where $a > b$. Find the length of a beam of uniform square cross section which can be cut from the tree trunk show that the beam has the greatest volume $\dfrac{8a^3 l}{27(a-b)}$	10
	OR	
(b)	Change the order of integration and evaluate $\int_0^1 dx \int_{y=1}^{\infty} e^{-y} y^x \log y\, dy$ with use of proper diagram the change on diagram.	10
2 **(a)**	State the stroke's theorem and verify this theorem for $F = xzi - yj + x^2 yk$, where S is the surface of the region bounded by x = 0, y = 0, z = 0 and 2x+y+2z = 8 which is not included on x-z plane	10
	OR	
(b)	Using the Legendre's homogeneous differential equation method solve the given differential equation $(3x+2)^2 \dfrac{d^2 y}{dx^2} - (3x+2)\dfrac{dy}{dx} - 12y = 6x$	10
3 **(a)**	Solve the given equations by Laplace transform $\dfrac{d^3 y}{dt^3} + 2\dfrac{d^2 y}{dt^2} - \dfrac{dy}{dt} - 2y = 0, where\ y = 1, \dfrac{dy}{dt} = 2\ at\ t = 0$	10
	OR	

(b)	**i.**	Find the Laplace transform of $te^{-4t}\sin 3t$		**10**
	ii.	Find the Laplace transform of $\displaystyle\int_0^t \frac{\sin t}{t}\,dt$		

PART - B (20 Marks)

4	A condenser of capacity C is charged through the inductance L and resistance K in series and the charge q at any time t satisfies the equation $L\dfrac{d^2 q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C} = 0$. Given that L= 0.25 henry, R=250 ohms, $C = 2\times10^6\ farad$ and that when t=0, the charge q is 0.002 coulombs, and current $\dfrac{dq}{dt} = 0$ obtain the value of q in terms of t.	**10**
5	Solve the given inverse Laplace transform by implementing convolution theorem $L^{-1}\left\{ \dfrac{s}{\left(s^2+1\right)\left(s^2+4\right)} \right\}$	**10**

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